Section 2.3
Matrix Multiplication

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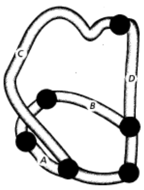
Investigation 3 - A Game of Touring

Two players compete in the computer game Touring. The object of the game is to travel along the paths to gain as many points as possible.



Here are the results for Amanda and Judy when they played Touring.

Path	A	B	C	D
Amanda	2	2	2	0
Judy	0	0	3	2

Path	Points
A	2
B	2
C	4
D	3

A. How many points did Amanda receive? ______________________

B. How many points did Judy receive? ______________________

C. Who won? ______________________

Questions:

1. To find Amanda's score, what numbers did you multiply? ______________

What numbers did you add? ______________________

2. To find Judy's score, what numbers did you multiply? ______________

What numbers did you add? ______________________

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3. Record the tables from Game 1 as well as your answer for the total scores in matrix form.

Number of Times Path Traveled

$$\begin{array}{cccc} A & B & C & D \\ \end{array}$$
$$\begin{pmatrix} 2 & 2 & \_ & \_ \\ 0 & \_ & \_ & \_ \end{pmatrix}$$

Points Earned for a Path

$$\begin{pmatrix} 2 \\ \_ \\ \_ \\ \_ \end{pmatrix}$$

Score Earned by Each Player

$$\begin{pmatrix} \_ \\ \_ \end{pmatrix}$$

4. Suppose Amanda and Judy travelled the paths in the same way as before, but the points awarded to each path were changed.

Path	A	B	C	D
Amanda	2	2	2	0
Judy	0	0	3	2

Path	Points
A	2
B	1
C	4
D	2

a) Use this information to complete three matrices as shown below.
Label each row.

Number of Times Path Traveled

$$\begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix}$$

Points Earned for a Path

$$\begin{pmatrix} \_ \\ \_ \\ \_ \\ \_ \end{pmatrix}$$

Score Earned by Each Player

$$\begin{pmatrix} \_ \\ \_ \end{pmatrix}$$

b) Who won? ____

5. If "Points Earned for a Path" matrices in Games 1 and 2 are combined into one matrix, it looks like this:

$$\begin{pmatrix} 2 & 2 \\ 2 & 1 \\ 4 & 4 \\ 3 & 2 \end{pmatrix}$$

What heading would go at the top of each column? ______________________________

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6. The information for Games 1 and 2 is given in the matrices below. Complete the third matrix.

Number of Times Path Traveled

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline \text{Amanda} & 2 & 2 & 2 & 0 \\ \text{Judy} & 0 & 0 & 3 & 2 \end{array}$$

Points Earned for a Path

$$\begin{array}{c|cc} & G.1 & G.2 \\ \hline A & 2 & 2 \\ B & 2 & 1 \\ C & 4 & 4 \\ D & 3 & 2 \end{array}$$

Score Earned by Each Player

$$\begin{array}{c|cc} & \square & \square \\ \hline \text{Amanda} & \_ & \_ \\ \text{Judy} & \_ & \_ \end{array}$$

a) What might the labels be at the top of each column in the third matrix?

____

b) There is a 14 in the third matrix. What does it represent?
In which row and which column is it located?

____

____

c) What numbers did you multiply and add to obtain the 14?
In which row and column are those numbers located?

____

____

d) There is an 18 in the third matrix. What does it represent?
In which row and in which column is it located?

____

____

e) What numbers did you multiply and add to obtain the 18?
In which row and which column are those numbers located?

____

____

f) Describe the pattern that tells you how to find the values in the third matrix based on the numbers in the other two matrices.

____

____

____

g) The dimensions of the first matrix is ___ rows and ___ columns.
The dimensions of the second matrix is ___ rows and ___ columns.
The dimensions of the third matrix is ___ rows and ___ columns.

h) What relationship(s) do you see among the number of rows and columns in each matrix?

____

____

____

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Review of Matrix Addition & Scalar multiplication

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Adding Matrices: Example

Air Nova and Provincial Air Lines (PAL) operate flights within Newfoundland. The tables below show the number of incoming and outgoing flights operated by each airline for St. John's, Gander and Deer Lake.

	Incoming flights		
	St. John's	Gander	Deer Lake
Air Nova	6	3	8
Provincial Air Lines	5	4	8

	Outgoing flights		
	St. John's	Gander	Deer Lake
Air Nova	6	4	7
Provincial Air Lines	6	4	8

The information in these tables can be represented by matrices.

Incoming Flights $\begin{pmatrix} 6 & 3 & 8 \\ 5 & 4 & 8 \end{pmatrix}$ Outgoing Flights $\begin{pmatrix} 6 & 4 & 7 \\ 6 & 4 & 8 \end{pmatrix}$

To calculate the total number of flights (incoming and outgoing) operated by both airlines, you could add the matrices

Incoming Flights, Outgoing Flights, All flights

$$\begin{pmatrix} 6 & 3 & 8 \\ 5 & 4 & 8 \end{pmatrix} + \begin{pmatrix} 6 & 4 & 7 \\ 6 & 4 & 8 \end{pmatrix} = \begin{pmatrix} 12 & 7 & 15 \\ 11 & 8 & 16 \end{pmatrix}$$

To add matrices, you combine each element in the first matrix with the corresponding element in the second matrix. For example, the element in row 1, column 2 of the first matrix (incoming flights) is 3. The corresponding element (row 1, column 2) in the second matrix (outgoing flights) is 4. When 3 and 4 are added, the result is 7, which is the element in row 1, column 2 of the third matrix (all flights).

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Scalar Multiplication: Example

Suppose there is a large upswing in the tourist industry in Newfoundland. Many people want to travel within the province by air. So Air Nova and Provincial Airlines decided to double the number of incoming flights which they have been operating in St. John's, Gander and Deer Lake.

Incoming flights | Double incoming flights

$$2 \times \begin{pmatrix} 6 & 3 & 8 \\ 5 & 4 & 8 \end{pmatrix} = \begin{pmatrix} 12 & 6 & 16 \\ 10 & 8 & 16 \end{pmatrix}$$

Here each element is the first matrix (incoming flights) is multiplied by 2 and product is placed in the corresponding row and column in the resulting matrix (double incoming flights).

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Matrix Multiplication

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Multiplying Matrices: Example

The table below shows the number of direct flights between St. John's, Gander and Deer Lake (operated by Air Nova and PAL).

		TO		
		St. John's	Gander	Deer Lake
FROM	St. John's	0	2	1
	Gander	1	0	1
	Deer Lake	2	1	0

The information in the table can be represented as a matrix.

$$\begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

To calculate how many one-stop flights there are between each location, you must square the matrix, that is, multiply the matrix by itself.

A, B, Product Matrix (A x B)

$$\begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 4 & 3 \end{pmatrix}$$

To multiply two matrices, you multiply each row by each column, placing your result in the corresponding row/column of the product matrix.

In the example above, the matrix A has 3 rows and 3 columns, and matrix B has 3 rows and 3 columns. Therefore, the product matrix will also have 3 rows and three columns.

Multiply row 1 in Matrix A by column 1 in matrix B ((0 x 0) + (2 x 1) + (1 x 2) = 4)

Place the solution (4) in row 1, column 1 of the product matrix. You continue in this manner until there are a total of 9 elements in the product matrix.

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Practice:

A, B, A x B

$$\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ 2 & 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} R1C1 & R1C2 & R1C3 & R1C4 \\ R2C1 & R2C2 & R2C3 & R2C4 \end{pmatrix}$$

$(3\times 0)+(1\times 2)+(1\times 1)$

$= 0 + 2 + 1$

$= 3$

$$= \begin{pmatrix} 1 & 4 & 4 & 1 \\ 5 & 10 & 3 & 5 \end{pmatrix}$$

1 4 4 1

5 10 3 5

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Rules: Matrix Multiplication

- Matrix A has 2 rows and 3 columns, matrix B has 3 rows and 4 columns, so the product matrix must have 2 rows and 4 columns.

- If the number of columns in the first matrix does not equal the number of rows in the second matrix, then the two matrices **cannot** be multiplied.

- Two square matrices (number of rows = number of columns) can be multiplied.

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$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 1 & 2 \\ 3 & 4 & 5 \end{pmatrix}$$

2×2 $\quad$ 2×3

$= 2 \times 3$

Worksheet

#1-6 State if the following can be multiplied.

-> dimensions of each matrix

-> what would be the dimensions of the answer matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 5 & 8 \\ 6 & 9 \\ 7 & 10 \end{pmatrix}$$

2×2 $\quad$ 3×2

cannot be mult.

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Classwork/Homework

(Questions from FMT text)

Pg. 21 #6

Pg. 22 #1

Pg. 25 #1a-g

If you are having trouble with multiplying matrices:
There are two good examples on Pg.24 & 25